Optimum Design of Stepped Transmission-Line Transformers*

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Summary—This paper describes the optimum stepped-transmission-line transformer structure for matching two unequal characteristic impedances. For any specified bandwidth, the steps are designed to yield a Tchebycheff-type (or equal-ripple) reflection-coefficient response. Over this band, the maximum vswr is less than that obtainable with any other stepped-transformer having the same number of steps. Design method and technique for eliminating discontinuity-capacitance effects are given. The measured results for a coaxial and a waveguide model are presented and found to verify the method.

Introduction

IN THIS PAPER a method of design will be given for a transformer structure that is capable of matching transmission lines of different characteristic impedances over a very broad band.[1] As shown in Fig. 1

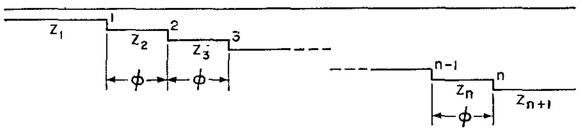

Fig. 1—The stepped transformer.

this structure consists of a succession of abrupt steps in characteristic impedance spaced by essentially equal electrical lengths of uniform line. The transmission line may be of any type, for example, coaxial or waveguide. With a specified number of steps, this design method provides the maximum possible bandwidth for a given vswr, or conversely, the minimum possible vswr for a given bandwidth. For this reason, the structure has been termed the *optimum*-stepped transformer. It may also be called the Tchebycheff transformer, since the Tchebycheff polynomial is used in its design.

Prior to this work, Hansen's *binomial-coefficient* design was the accepted method for the stepped transformer.[2] In this design, the logarithms of the impedance ratios of the steps are made to be in the ratio of the binomial-coefficients; i.e., in the ratio of numerical coefficients of $(x+y)^{n-1}$, where n is the number of steps. Subject to Hansen's assumptions of small steps, zero discontinuity capacitance, and equal electrical lengths between steps, theoretical vswr of binomial transformer is

$$S = 1 + (\cos\phi)^{n-1} \ln \frac{Z_{n+1}}{Z_1}, \qquad (1)$$

where ϕ is the electrical phase length between steps, n is the number of steps, Z_{n+1} is the characteristic impedance of the higher-impedance terminating line, and Z_1 is that of the lower-impedance terminating line.

The improved method of design to be described in this paper proportions the logarithms of the step ratio in such a manner that the vswr has the characteristic "equal-ripple" response of a Tchebycheff polynomial. Subject to the same approximations assumed for the binomial-coefficient design, and for a specified number of steps, it can be shown analytically that the Tchebycheff design gives the maximum possible bandwidth for a given vswr, or the minimum possible vswr for a given bandwidth. The degree of improvement is evident in Fig. 2, where the theoretical vswr is plotted as a function of the step spacing for five-step transformers having the binomial and the optimum responses. In this example, the optimum transformer was designed for a two-to-one band, and has a maximum vswr of 1.021 in this range. The binomial design has a vswr of 1.13 at the edges of this range, and its bandwidth for a vswr of 1.021 is only 1.52 to one. The same sort of improvement will occur for any number of steps and for any bandwidth however small or large, as long as the optimum transformer is designed for that particular bandwidth.

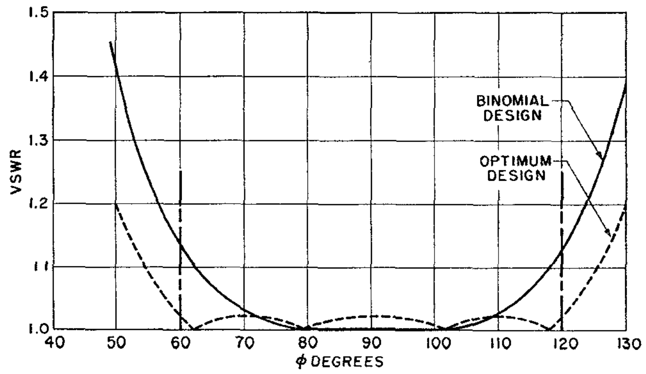

Fig. 2—Theoretical vswr response of a five-step transformer for a total characteristic impedance change of 8:1.

* This work was performed in 1951 at the Sperry Gyroscope Co., New York, N. Y., and is described in part in the Third and Fourth Quarterly Reports on Development of Broadband Waveguide Components, April 20, and July 20, 1951. The program was supported by the Signal Corps under Contract No. DA-36-039-sc-166.

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[1] The writer has learned recently of independent work on optimum-stepped transformers by: F. Bolinder, "Fourier transforms in the theory of inhomogeneous transmission lines," *Acta Polytech. Elec. Engrg. Ser.* (*Stockholm*), vol. 3, pp. 3–84, 88; 1951. H. J. Riblet, "Optimum (Narrow Band) Transformer and Directional Coupler Performance," presented at URSI Meeting, Washington, D. C.; April 28, 1953. S. Hopfer, "Techniques Utilizing Flat and Ridged Waveguides," presented at Symposium on Modern Advances in Microwave Techniques, New York, N. Y.; November 10, 1954. It is likely that others may also have made this extension of Dolph-Tchebycheff antenna-array theory to transmission-line transformers. R. E. Collin "Theory and Design of wide-band multi-section quarter-wave transformers" Proc. IRE, vol. 43, pp. 179–185; February, 1955.

[2] W. W. Hansen, "Notes on Lectures," ch. 6; M. I. T. Rad. Lab., 1941–1944.

Design Relations

The design method given in this paper is similar to one developed by Dolph[3] for antenna arrays. When applied to the stepped-transformer, the voltage-standing-wave ratio is

$$S = 1 + \ln \left[\frac{Z_{n+1}}{Z_1}\right] \cdot \frac{T_{n-1}\left[\dfrac{\cos \phi}{\cos \phi_1}\right]}{T_{n-1}\left[\dfrac{1}{\cos \phi_1}\right]}, \tag{2}$$

where ϕ_1 is the electrical spacing of the steps at the low-frequency edge of the band and $T_m(x)$ is the Tchebycheff polynomial of mth degree defined by

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1 \\ &\cdots \\ T_{m+1}(x) &= 2xT_m(x) - T_{m-1}(x) \\ &\cdots \end{aligned} \tag{3}$$

$T_m(x)$ may also be computed from the following equivalent expressions

$$T_m(x) = \cos(m \cos^{-1} x), \qquad |x| \leqq 1 \tag{4a}$$

$$T_m(x) = \cosh(m \cosh^{-1} x), \qquad |x| \geqq 1. \tag{4b}$$

The maximum vswr in the design band is

$$S_{\max} = 1 + \frac{\ln \left[\dfrac{Z_{n+1}}{Z_1}\right]}{T_{n-1}\left[\dfrac{1}{\cos \phi_1}\right]}. \tag{5}$$

Eqs. (2) and (5), which are derived in Appendix I, are valid subject to the assumption of small steps in impedance, but as will be shown later by an example, they hold quite well even for surprisingly large steps.

In terms of the electrical lengths ϕ_1 and $\phi_2 = 180$ degrees$-\phi_1$ at the band edges, the bandwidth ratio p is given in coaxial line by

$$p = \frac{f_2}{f_1} = \frac{\phi_2}{\phi_1} = \frac{180 \text{ degrees} - \phi_1}{\phi_1} \tag{6}$$

and in waveguide by

$$p = \frac{\phi_2}{\phi_1} = \frac{\lambda_{g1}}{\lambda_{g2}}. \tag{7}$$

For a desired value of p, ϕ_1 may be obtained from

$$\phi_1 = \frac{180 \text{ degrees}}{1 + p}. \tag{8}$$

[3] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beam width and side-lobe level," Proc. IRE, vol. 34, pp. 335-348; June, 1946.

In coaxial line, the step-spacing is a quarter wavelength at the center frequency of the band, or

$$L = \frac{\lambda_0}{4} = \frac{\lambda_1 \lambda_2}{2(\lambda_1 + \lambda_2)}. \tag{9}$$

In waveguide

$$L = \frac{\lambda_{g0}}{4} = \frac{\lambda_{g1} \lambda_{g2}}{2(\lambda_{g1} + \lambda_{g2})}. \tag{10}$$

In order to obtain the vswr given by (2), the ratios of the reflection coefficients of the steps must be equal to the ratios of a certain set of constants a_m which may be computed for the particular number of steps and the desired bandwidth:

$$r_1 : r_2 : r_3 : \cdots : r_n = a_1 : a_2 : a_3 : \cdots : a_n. \tag{11}$$

In order to simplify the computation and make possible an explicit formula for the characteristic impedances, the following approximation proposed by Hansen will be used

$$r_m = \frac{1}{2} \ln \frac{Z_{m+1}}{Z_m}, \qquad r_m < \frac{1}{3}. \tag{12}$$

This agrees within a few per cent with the exact formula for r even for Z_{m+1}/Z_m as large as 2.0. It should not be assumed, however, that the inaccuracy of (12) is the sole theoretical factor limiting the design method to small steps. Other factors are the reflection interactions between large steps and the diminution of the transmitted wave at large steps. With the use of (12), the step ratios may be given by

$$\ln \frac{Z_2}{Z_1} : \ln \frac{Z_3}{Z_2} : \cdots : \ln \frac{Z_{n+1}}{Z_n} = a_1 : a_2 : a_3 : \cdots : a_n. \tag{13}$$

The ratio at a given step may be computed in terms of the terminating impedances and the a_m values by

$$\ln \frac{Z_{m+1}}{Z_m} = \frac{a_m \ln \dfrac{Z_{n+1}}{Z_1}}{a_1 + a_2 + a_3 + \cdots + a_n}. \tag{14}$$

Once Z_{m+1}/Z_m is known at each step, the characteristic impedance of each section of the transformer may be obtained.

A simple method for calculating the a_m values necessary in (14) is given in Appendix II. Also, as a further aid to the design engineer, values of a_m are tabulated (Tables I, II and III, page 18) for bandwidth ratios of 1.40, 2.00, and 2.27, and for various numbers of steps.

The assumption of small steps would appear to limit the utility of both the Tchebycheff and binomial design methods in the case of a large impedance change. In order to determine the effect of a large violation of the small-step assumption, the hypothetical case of a 5-step transformer having $p = 2$ and $Z_6/Z_1 = 8$ was investigated. As shown in Fig. 2, the vswr for this case reaches the maximum value of 1.021 at five points in the band. Due

TABLE I

n	a_m Values for $p=1.40$
1	1
2	1, 1
3	1, 1.8661, 1
4	1, 2.799, 2.799, 1

TABLE II

n	a_m Values for $p=2.00$
1	1
2	1, 1
3	1, $1\frac{1}{2}$, 1
4	1, $2\frac{1}{4}$, $2\frac{1}{4}$, 1
5	1, 3, $4\frac{1}{8}$, 3, 1

TABLE III

n	a_m Values for $p=2.27$
5	1, 2.684, 3.585, 2.684, 1
7	1, 4.026, 8.078, 10.033, 8.078, 4.026, 1

to symmetry, however, only three of these points need be considered. The various characteristic impedances of the transformer were determined, and then the input vswr of the terminated transformer was computed by exact methods at the three critical values of ϕ. The vswr's thus obtained were 1.025, 1.014, and 1.027. These values compare very well with the value of 1.021 determined from (5). Therefore, it appears that the small-step assumption may be violated drastically without excessive deterioration in the performance of the transformer.

Correction for Discontinuity Susceptances

It has been assumed thus far that the discontinuity susceptances in the stepped transformer are zero. This would be approximately true in a low-frequency coaxial line, but not in a high-frequency coaxial line or waveguide. The presence of the discontinuity susceptances has two effects. The lesser effect is a small increase in the magnitudes of the individual step reflections. The greater effect is the introduction of phase angles in the reflection and transmission coefficients of the steps. In the following analysis these effects will be investigated, and methods of correction for these effects will be given.

Fig. 3 shows the equivalent circuit of a single step in an otherwise infinite transmission line. The voltage reflection and transmission coefficients of the step are:

$$r_m=\frac{Y_m-Y_{m+1}-jB_m}{Y_m+Y_{m+1}+jB_m}=\frac{Y_m/Y_{m+1}-1-jB_m/Y_{m+1}}{Y_m/Y_{m+1}+1+jB_m/Y_{m+1}} \tag{15}$$

$$t_m=\frac{2Y_m/Y_{m+1}}{Y_m/Y_{m+1}+1+jB_m/Y_{m+1}}. \tag{16}$$

The magnitudes of r_m and t_m are

$$|r_m| = \sqrt{\frac{(Y_m/Y_{m+1}-1)^2+(B_m/Y_{m+1})^2}{(Y_m/Y_{m+1}+1)^2+(B_m/Y_{m+1})^2}} \tag{17}$$

$$|t_m| = \sqrt{\frac{(2Y_m/Y_{m+1})^2}{(Y_m/Y_{m+1}+1)^2+(B_m/Y_{m+1})^2}}. \tag{18}$$

It is seen that r_m is not greatly affected by B_m, if $(B_m/Y_{m+1})^2 \ll (Y_m/Y_{m+1}-1)^2$. This would be the case in a low-frequency coaxial line, and would be approximately the case in waveguide. If this condition is not met, the step impedances should be re-computed by successive approximations until the r_m values for $m=1$ to n are in the required ratio $a_1:a_2:a_3: \cdots :a_n$. The effect of B_m on t_m is even less, and is likely to be negligible in any case. The phase angles of r_m and t_m are

$$\angle r_m = -\tan^{-1}\left(\frac{B_m/Y_{m+1}}{Y_m/Y_{m+1}-1}\right) - \tan^{-1}\left(\frac{B_m/Y_{m+1}}{Y_m/Y_{m+1}+1}\right) \tag{19}$$

$$\angle t_m = -\tan^{-1}\left(\frac{B_m/Y_{m+1}}{Y_m/Y_{m+1}+1}\right). \tag{20}$$

By interchanging Y_m and Y_{m+1} it may be seen that $\angle t_m$ is independent of the direction of transmission.

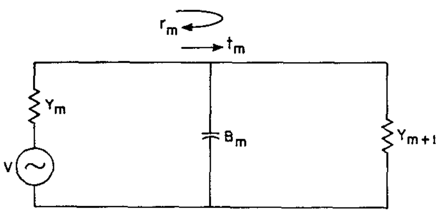

Fig. 3—Equivalent circuit of a single step.

Now consider the individual waves reflected from the steps arriving at some particular reference point between the generator and the transformer. It will be found that the phase angles of the reflection and transmission coefficients cause phase shifts in these returning waves in addition to those due to the distance traversed. These extra phase shifts are

$$\begin{aligned}\phi_{e1} &= -\angle r_1\\ \phi_{e2} &= -\angle r_2 - 2\angle t_1\\ &\cdots\cdots\cdots\\ \phi_{en} &= -\angle r_n - 2\angle t_1 - 2\angle t_2 \cdots - 2\angle t_{n-1}.\end{aligned} \tag{21}$$

At the center of the band ($\phi=90$ degrees) these extra phase shifts may be eliminated by moving each step toward the generator by an electrical length equal to one-half of the extra phase shift. The distance x by which each step is moved is therefore as follows:

$$x_1 = \frac{\phi_{e1}}{2\beta}$$

$$x_2 = \frac{\phi_{e2}}{2\beta} \tag{22}$$

$$\cdots$$

$$x_n = \frac{\phi_{en}}{2\beta},$$

where $\beta = 360$ degrees/λ_g in wave guide and 360 degrees/λ in coaxial (or other TEM-mode) line. Although this correction is made only at the center of the band, it should give good results over a wide range in ϕ. The physical result of this correction is in most cases to decrease the spacings between steps to somewhat less than $\lambda/4$ (or $\lambda_g/4$).

Ample theoretical data to permit accurate computation of the above correction exists for the discontinuity susceptance of steps in coaxial, parallel-plane, and waveguide lines.[4,5]

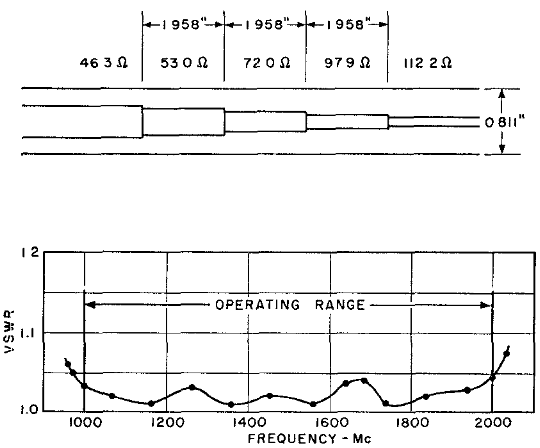

Fig. 4—The stepped coaxial transformer and its measured vswr.

Experimental Tests

Data is at present available on two optimum-step transformers. The first is a four-step coaxial transformer with $p = 2.0$, and a design range of 1,000 to 2,000 mc. As shown in Fig. 4, one end of the transformer connects to standard 7/8-inch line having a characteristic impedance of 46.3 ohms, while the other connects to a line having the same O.D., but a characteristic impedance of 112.2 ohms. The lengths were calculated from (9) and the characteristic impedances from (14) with the use of the a_m values of Table II for $n = 4$. The discontinuity susceptance corrections were computed by (19) to (22) and were found to require a shortening of each length by about 0.5 per cent, a quantity small enough to be neglected in this case. The maximum vswr computed from (5) is 1.034, while the measured vswr in the band has a maximum of 1.045. The difference between these values is very small and may be due to test-equipment errors. For the same number of steps, (1) shows that a binomial-coefficient design would have a maximum vswr of 1.11 in the band.

The second transformer was constructed in 2×1-inch waveguide for use with a waveguide filter. In this application, it was necessary to transform from the standard height of 0.872 inches to a height of 0.550 inches, with the width dimension held constant. The required frequency range is 4,400 to 5,200 mc, yielding a value of $p = \lambda_{g1}/\lambda_{g2} = 1.35$, but to provide some tolerance the value $p = 1.40$ was used in the design. It was found by (5) that a theoretical vswr of 1.016 could be held over the band for $n = 3$ and therefore this number of steps was selected. In designing the impedance levels in the transformer, use was made of the fact that for a constant width, the characteristic impedance of a waveguide is proportional to its height. Upon calculating the discontinuity-susceptance effect, it was found that the change in the magnitude of the reflection coefficients was negligible, but that a substantial change in one of the section lengths was required. The final dimensions are shown in Fig. 5. The measured vswr, also shown in Fig. 5, has a maximum value of 1.045 in the band. Since the theoretical vswr is actually less than the nominal accuracy of the test equipment, part of the additional reflection may be due to experimental error. In any case, the performance obtained with this and the coaxial transformer is considered to be a satisfactory verification of the design method.

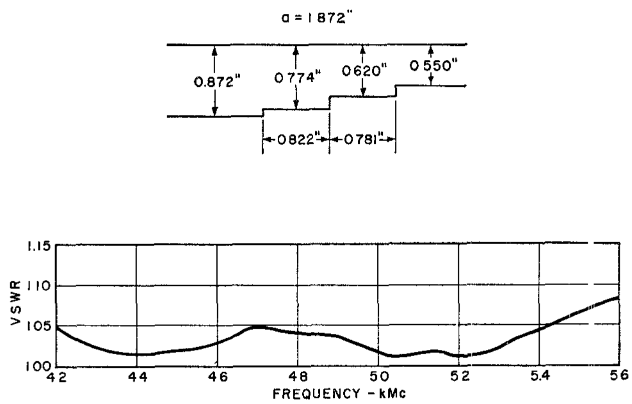

Fig. 5—The stepped waveguide transformer and its measured vswr.

Conclusion

It has been shown theoretically and experimentally that the optimum-stepped transformer is superior to the previously used binomial transformer. Since the former is no more difficult to design or construct, it is recommended that it be used in all future applications requiring a stepped transformer.

The discontinuity-susceptance correction, which could also be used in the binomial transformer, makes

[4] J. R. Whinnery, H. W. Jamieson, and T. E. Robbins, "Coaxial line discontinuities," Proc. IRE, vol. 32, pp. 697–709; November, 1944.

[5] N. Marcuvitz, "The Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N. Y.; 1951.

possible the successful design of stepped transformers in waveguide, where discontinuity effects have been generally troublesome in the past.

Appendix I

Derivation of the Design Formulas

With reference to Fig. 1, the voltage reflection coefficient of the mth step is

$$A_m = \frac{Z_{m+1} - Z_m}{Z_{m+1} + Z_m}. \tag{23}$$

Because of the assumption that the steps are small, we may neglect reflection interactions, and express the total reflection coefficient of the stepped transformer referred to the center as follows

$$\rho = A_1 e^{j(n-1)\phi} + A_2 e^{j(n-3)\phi} + A_3 e^{j(n-5)\phi} + \cdots + A_n e^{-j(n-1)\phi}. \tag{24}$$

The step reflections are assumed to be symmetrical; i.e., $A_1 = A_n$, $A_2 = A_{n-1}$, etc. Therefore, for n odd

$$\rho = 2A_1 \cos(n-1)\phi + 2A_2 \cos(n-3)\phi \cdots + A_{(n+1)/2} \tag{25}$$

and for n even

$$\rho = 2A_1 \cos(n-1)\phi + 2A_2 \cos(n-3)\phi + \cdots + 2A_{n/2} \cos\phi. \tag{26}$$

In order to obtain optimum performance, the coefficients A_m must be chosen so that ρ will be proportional to a Tchebycheff polynomial. These polynomials are defined in (3) and (4) of this paper. A study of their properties shows that they all oscillate between ± 1 for x between ± 1. For $|x| > 1$, $|T_m(x)|$ increases monotonically.

To obtain the desired reflection-coefficient response proceed as follows. First substitute the following trigonometric identities in (25) and (26).

$$\begin{aligned} \cos\phi &= w \\ \cos 2\phi &= 2w^2 - 1 \\ \cos 3\phi &= 4w^3 - 3w \\ \cos 4\phi &= 8w^4 - 8w^2 + 1 \\ &\text{etc.} \end{aligned} \tag{27}$$

Then set

$$w = \cos\phi = x \cos\phi_1. \tag{28}$$

Note that this last relation makes $x \leqq 1$ for ϕ between ϕ_1 and ϕ_2, where $\phi_2 = 180$ degrees $- \phi_1$. It is this symmetrical range of ϕ in which the reflection coefficient is desired to remain low. The angle ϕ_1 may be chosen to be any value between zero and 90 degrees. For example, if $\phi_1 = 60$ degrees, then $\phi_2 = 120$ degrees, and $\phi_2/\phi_1 = 2$.

After (27) and (28) are substituted in (25) or (26), a polynomial of either odd or even powers of x is obtained. For a transformer having n steps, the highest power is $n-1$. The resulting polynomial may then be set equal to $\alpha T_{(n-1)}(x)$ in order to determine the coefficients A_m. α is a constant of proportionality that will be evaluated later.

The following ratios are determined by the above procedure:

$$A_1 : A_2 : A_3 : \cdots : A_n = a_1 : a_2 : a_3 : \cdots a_n, \tag{29}$$

where $a_m = A_m / A_1$. The various characteristic impedances of the transformer may be readily computed from these reflection-coefficient ratios by means of (14) of this report.

Once the characteristic impedances have been properly assigned, the reflection coefficient of the transformer is given by

$$\rho = \alpha T_{(n-1)}(x). \tag{30}$$

But x is related to ϕ by

$$x = \frac{\cos\phi}{\cos\phi_1} \tag{31}$$

and, therefore,

$$\rho = \alpha T_{(n-1)}\left(\frac{\cos\phi}{\cos\phi_1}\right). \tag{32}$$

Subject to the assumption of small steps, ρ is $\ll 1$, and therefore the voltage-standing-wave ratio is

$$S = 1 + 2\alpha T_{(n-1)}\left(\frac{\cos\phi}{\cos\phi_1}\right). \tag{33}$$

The constant α may be determined from a knowledge of ρ for $\phi = 0$. In this case, the reflection coefficient is approximately equal to the sum of the individual step reflections, and therefore is given by

$$\rho\,|_{\phi=0} = \frac{1}{2} \ln \frac{Z_{n+1}}{Z_1}. \tag{34}$$

When this is substituted in equation (32) one obtains

$$\alpha = \left(\frac{\rho}{T_{(n-1)}\left(\frac{\cos\phi}{\cos\phi_1}\right)} \right)_{\phi=0} = \frac{\frac{1}{2}\ln\frac{Z_{n+1}}{Z_1}}{T_{(n-1)}\left(\frac{1}{\cos\phi_1}\right)} \tag{35}$$

and, hence, the formulas for ρ and S are

$$\rho = \frac{1}{2} \ln\left(\frac{Z_{n+1}}{Z_1}\right) \frac{T_{(n-1)}\left(\frac{\cos\phi}{\cos\phi_1}\right)}{T_{(n-1)}\left(\frac{1}{\cos\phi_1}\right)} \tag{36}$$

$$S = 1 + \ln\left(\frac{Z_{n+1}}{Z_1}\right) \frac{T_{(n-1)}\left(\frac{\cos\phi}{\cos\phi_1}\right)}{T_{(n-1)}\left(\frac{1}{\cos\phi_1}\right)}. \tag{37}$$

It should be noted that the use of equation (34) for $\phi = 0$

in (36) and (37) leads to more accurate results in and near the operating band of the transformer than would the exact formula for $\rho|_{\phi=0}$.

Appendix II

The following simplified method of calculating the a_m values was developed for antenna-array applications by Ross E. Graves in an as yet unpublished report. It is adapted here with his permission for the stepped-transformer case.

TABLE IV
Computation of Relative a_m Values for $p=1.40$

$n=1$	2					
$n=2$		3.864				
$n=3$	27.861		14.930			
$n=4$		161.48		57.690		
etc.						

To employ Graves' method, it is necessary to construct a numerical table by a simple recursion procedure. To illustrate the method, a typical table is given above in Table IV for the case of $p=1.40$, $\phi_1=75.0$ degrees. In the upper left-hand corner always insert the number two for any value of p. In the second column, second row, always insert

$$x_0 = \frac{1}{\cos \phi_1}.$$

For this example, $x_0=1/\cos 75$ degrees$=3.864$. Then fill in the table by means of the following rules until the desired value of n is reached.

1. To find an additional entry in the first column, multiply the element on the right just above by $2x_0$ and then subtract the element in the second row above the entry to be found.
2. To find an additional entry in any other column, add the two elements on the left and right just above and multiply by x_0, and then subtract the element in the second row above the entry to be found.
3. Where an element is absent, assume it to be zero.

The illustrative table has been filled up to $n=4$. The elements in the table are in the ratio of the a_m constants, the first column corresponding to the center of the transformer. For example, for $n=3$,

$$a_1:a_2:a_3 = 14.930:27.861:14.930 = 1:1.8661:1$$

and for $n=4$,

$$a_1:a_2:a_3:a_4 = 57.690:161.48:161.48:57.690$$
$$= 1:2.799:2.799:1.$$

The table could be carried, if desired, to any value of n, no matter how large.

The Use of Scattering Matrices in Microwave Circuits

E. W. MATTHEWS, JR.†

Summary—**Difficulties arising from the use of the impedance concept in microwave circuitry have led to the introduction of the scattering representation for work at these frequencies. This paper presents a development of the scattering approach in terms of fundamental transmission-line phenomena. The physical meaning of the quantities involved is brought out wherever possible and the relationships among the various elements of the scattering matrix are given. Several examples of the application of scattering techniques to analysis of the properties of microwave junctions are presented, and methods for measuring scattering parameters of such junctions are outlined.**

Introduction

In conventional circuit theory, the fundamental quantities of interest are voltages and currents, and the parameters used to express relationships between them are called impedances or admittances. A single two-terminal circuit element may be characterized by a complex impedance, representing the ratio between the voltage and the current at its two terminals. The real part of this impedance (resistance) is related to the power dissipated in the circuit element, while the imaginary part (reactance) is a measure of the average energy stored in the element.

More complicated multi-terminal networks may be represented at a given frequency by an "equivalent circuit" consisting of a number of simple two-terminal elements in certain combinations or configurations, such as equivalent tee, pi, or ladder networks. The properties of such networks may alternatively be described in terms of generalized impedance (or admittance) relationships between terminals (or "ports," as currently named). This description is better understood generally in terms of the "self" and "mutual" impedances commonly used in coupled-circuit analysis as well as the "transfer" impedances appearing in vacuum-tube circuitry.

At microwave frequencies, certain difficulties are encountered in the application of conventional low-frequency circuit analysis techniques. As circuit dimen-

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